General Physics II: Midterm Exam (114–2)

100 minutes, full mark = 50

It is **strictly forbidden** to use your notebooks/memos/books.
It is **strictly forbidden** to **have/wear/use** any electric devices, even in your pockets.
It is **strictly forbidden** to discuss with other students.

Administrative Remarks

- Use full-sized sheets for your answers. Smaller sheets are for calculation, not to submit.
- Write your name and student ID on the answer sheet. Put your student ID on the desk.
- Allowed on your desk: student ID card (required), pens/pencils, correction tools (eraser etc.), rulers, and drinks. **Other items must be stored in your bags.**
- You cannot wear watches nor electronic devices. **You cannot have them even in your pockets**.
- **After 13:10, the following actions are considered cheating. You may immediately lose your credit.**
  - If non-allowed items (pen cases, foods, poaches, etc.) are found on desks.
  - If you have textbooks, mobile phones, tablets, or PC, if they are not stored in your bags, or if you use them. They must be in your bags even after you submit your answer sheets.
- Breaks are not allowed in principle. After 14:00, you may leave after submission. In case of health problems or other issues, call the TA or lecturer.
- *Any form of academic dishonesty, including chats, additions/corrections after the period, and using your phones, will be treated by NSYSU "Academic Regulations."*

Scientific Remarks

- Show your calculations or thought process for **partial mark**!
- Use English, where mistakes are tolerated. Meanwhile, scientific mistakes are not tolerated.
  - Provide appropriate **units** properly.
  - Clearly distinguish **vectors** (by writing $\vec{E}$, $\vec{x}$ or $\mathbb{E}$, $\mathbb{x}$) from scalars (E, x).
- If you find any errors or issues in the questions, explain them on your answer sheet, make necessary adjustments on the question, and answer accordingly.
- You may use the following symbols and values without definition/declaration.

standard acceleration of gravity	g	$= 9.8\,\mathrm{m/s^2}$
elementary charge	e (or $\lvert e\rvert$)	$= 1.6\times10^{-19}\,\mathrm{C}$
permittivity of free space	ϵ_0	$= 8.9\times10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$
permeability of free space	$\mu_0 = \left(\epsilon_0 c^2\right)^{-1}$	$= \pi\times 4.0\times10^{-7}\,\mathrm{N/A^2}$
Coulomb constant	$k_e = \left(4\pi\epsilon_0\right)^{-1}$	$= 9.0\times10^{9}\,\mathrm{N\cdot m^2/C^2}$
speed of light in vacuum	c	$= 3.0\times10^{8}\,\mathrm{m/s}$
Avogadro's number	N_A	$= 6.0\times10^{23}/\mathrm{mol}$
masses of protons and electrons	m_p, m_e	$= 1.7\times10^{-27}\,\mathrm{kg}, 9.1\times10^{-31}\,\mathrm{kg}$

Unit vectors in the direction of the axes $(\vec{e_x}, \vec{e_y}, \vec{e_z})$ or $(\hat{e_x}, \hat{e_y}, \hat{e_z})$ or $(\hat{\imath}, \hat{\jmath}, \hat{k})$

$\vec{E}(\vec{x})$	electric field at $\vec{x}$	$\vec{B}(\vec{x})$	magnetic field (magnetic flux density) at $\vec{x}$
$V(\vec{x})$	electrostatic potential at $\vec{x}$	$\vec{F}_{XY}$	force exerted by X on Y

$\sqrt{2}\approx 1.414$ $\sqrt{3}\approx 1.732$ $\sqrt{5}\approx 2.236$ $\sqrt{7}\approx 2.646$ $\pi\approx 3.142$ $\mathrm{e}\approx 2.718$

Answer all the problems.

[Part I] Fundamental Concepts (15 points)

Read the following text and answer the questions below.

Electrical **(A)** are materials in which all electrons are bound to atoms. In contrast, copper and aluminum are conductors. In a conductor, electric charges distribute only on the **(B)** of the material, and electric current can flow easily through it.

Microscopically, electric current I in a conductor is described by the equation $I = n|q|v_\mathrm{d}A$, where n is the number density of **(C)** and q is the electric charge of each of them. We call v_d the **(D)**, and A is the cross-sectional area of the conductor. If we use the current density $\vec{J} = nq\vec{v}_\mathrm{d}$, the current is given by $I = |\vec{J}|A$.

In this lecture course, we discussed the microscopic description of circuit elements and obtained several formulae, such as

$$\vec{J} = \sigma\vec{E}, \qquad R = \frac{\Delta V}{I}, \qquad R = \frac{\rho\ell}{A} = \frac{\ell}{\sigma A}, \qquad P = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}.$$

Here, R is the **(E)** of the conductor, ℓ is its length, **(F)** is the resistivity of the conductor material, **(G)** is the conductivity of the conductor material, and so on.

(1) Write the appropriate words or symbols for each blank.

(2) Write the unit of each of the following symbols appearing in the above text.

A) n **B)** $\vec{J}$ **C)** v_d **D)** ρ **E)** ΔV **F)** P

(3) Describe the definition of $\vec{v}_\mathrm{d}$ with using the phrase "charge carrier", paying attention that it is a vector quantity.

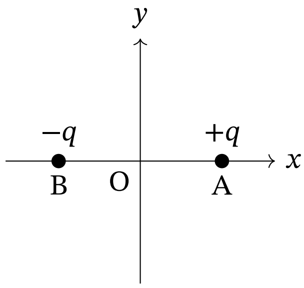
[Part II] Coulomb's law and Electric field (14 points)

Consider three points A$(h, 0, 0)$, B$(-h, 0, 0)$, and P(a, b, c) in three-dimensional space, where $h > 0$.

(1) Express $|\overrightarrow{\mathrm{OP}}|$, $|\overrightarrow{\mathrm{AP}}|$, and $\overrightarrow{\mathrm{OA}} \cdot \overrightarrow{\mathrm{OP}}$ by using a, b, c, h.

(2) Let $\vec{e}$ be the unit vector in the direction of $\overrightarrow{\mathrm{AP}}$. Express $\vec{e}$ by using a, b, c, h.

As in the figure on the right, place a positive charge $+q$ at Point A and a negative charge $-q$ at Point B.



(3) What is this configuration called?

(4) Find the force exerted on the positive charge A.

(5) Draw (sketch) the electric field lines.

(6) We assume the reference level of potential is $V = V_0$ at infinity. Find the electric field and the electric potential at Point O.

[The exam questions continue on the next page.]

[Part III] Capacitors (9 points)

Modern smartphones use memory implemented with DRAM (dynamic random-access memory). DRAM stores information using capacitors; a 12 GB memory contains about 10^{11} capacitors. For simplicity, consider a DRAM consisting of 1.0×10^{11} identical capacitors. Assume each capacitor is a parallel-plate capacitor with capacitance 32 fF, where $1\,\text{fF} = 1 \times 10^{-15}$ F.

First, focus on one capacitor in the DRAM. Initially, it is uncharged and the voltage difference ΔV between the plates is zero. Then, a voltage of 1.0 V is applied and the plates get charges $\pm Q$.

(1) Find the charge Q stored in the single capacitor.

(2) Estimate the number of electrons that move onto the negatively charged plate.

(3) Find the energy stored in the single capacitor.

Now consider a DRAM chip with dimensions 133.33 mm × 30.0 mm × 1.5 mm. Suppose all 1.0×10^{-11} capacitors are placed in a single layer. Then, the area available for each capacitor is, maximally, $133.33\,\text{mm} \times 30.0\,\text{mm} \div (1.0 \times 10^{11}) =$ ☐ nm^2.

(4) Recalling $1\,\text{nm} = 10^{-9}$ m, find the number for the above blank.

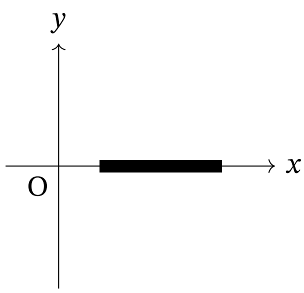
(5) Assume each capacitor is a parallel-plate capacitor with capacitance of 32 fF, the plates have an area of $(1000\,\text{nm})^2$ [Note: This is different from the value obtained above.], and the gap is filled with air. Estimate the plate separation.

(6) Since atoms typically have a size on the order of 10^{-10} m, this simple model is unrealistic. How can we overcome this problem? Suggest your ideas as many as possible.

[Part IV] Continuous Charge Distribution (12 points)

You may use the following values: $\ln 2 \approx 0.693$, $\ln 3 \approx 1.01$, $\ln 5 \approx 1.61$, $\ln 7 \approx 1.95$.

As shown in the figure on the right, a rod is placed along the x-axis. The rod, made of an insulator, has length $l = 4.0$ m and the distance between Point O and its left end is $a = 1.0$ m. Namely, its left end is at $(x, y) = (1.0\,\text{m}, 0)$ and its right end is at $(5.0\,\text{m}, 0)$. The rod has a uniform charge distribution with a total charge $Q = 2.0 \times 10^{-6}$ C.



(1) Find the linear charge density of the rod.

(2) Find the electric field at Point O.

(3) Find the electric potential at Point O, assuming the reference level of potential is $V = 0$ at infinity.

(4) Find the electric field at $(x, y) = (1.5\,\text{m}, 1.0 \times 10^{-3}\,\text{m})$ and at $(1.5\,\text{m}, 1.0 \times 10^{3}\,\text{m})$, assuming that the rod is very thin, or in other words, its transverse dimension is much less than 1 mm.

[The exam questions continue on the next page.]

An extra problem is given below. You may try writing something for a partial mark, because no penalty is given for incorrect answer as long as you are a student.

[Part V] Extra Problem (unlimited points)

(1) Consider the set up given in **[Part I]** of this exam. Find the electric field and the electrostatic potential at general point (x, y, z).

(2) Consider the set up given in **[Part IV]** of this exam. Find the electric field and the electrostatic potential at general point (x, y, z). You may answer with Q, l, and a insetad of the numbers.

[This is the end of the exam questions.]