Term Exam 112–1 (Dec. 20, 2023)

100 minutes, full mark = 50

Use of your notebooks/memos/books:	You can bring-in only a sheet of A4 paper.
Use of your mobile etc. & Internet:	Strictly forbidden.
Discussion with other attending students:	Strictly forbidden.

Administrative Remarks

- Write your name and student ID on the answer sheet. Put your student ID on the desk.
- Allowed on your desk: student ID card (required), pens/pencils, correction tools (eraser etc.), rulers, drinks, and the brought-in A4 sheet. **Other items must be stored in your bags.**
- **After 9:20, the following acts are regarded as cheating. You may immediately lose your credit.**
  - If non-allowed items (pen cases, foods, poaches, etc.) are found on desks.
  - If your mobile phones, tablets, or PC are not stored in your bags, or if you use them. They must be in your bags even after you submit your answer sheets.
- Breaks are not allowed in principle. After 10:10, you may leave after submission. In case of health problems or other issues, call the TA or lecturer.
- *Any form of academic dishonesty, including chats, additions/corrections after the period, and using your phones, will be treated by NSYSU "Academic Regulations."*

Scientific Remarks

- Include your calculations or thinking process for **partial mark**!
- Use English, where mistakes are torelated. Meanwhile, scientific mistakes are not tolerated.
  - Provide appropriate **units** if necessary.
  - Handle **significant digits** properly.
  - Clearly distinguish **vectors** (by writing $\vec{E}$, $\vec{x}$ or $\mathbb{E}$, $\mathbb{x}$) from scalars (E, x).
- If you notice any errors/issues in the questions, explain the error on your answer sheet, suitably adjust the question, and answer the corrected question.
- You may use the following symbols and values without definition/declaration.

standard acceleration of gravity	g	$= 9.8\,\mathrm{m/s^2}$
Newtonian constant of gravitation	G	$= 6.7\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}$
elementary charge	e (or $\lvert e\rvert$)	$= 1.6\times10^{-19}\,\mathrm{C}$
permittivity of free space	ϵ_0	$= 8.9\times10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$
permeability of free space	$\mu_0 = \left(\epsilon_0 c^2\right)^{-1}$	$= 1.3\times10^{-6}\,\mathrm{N/A^2}$
Coulomb constant	$k_e = \left(4\pi\epsilon_0\right)^{-1}$	$= 9.0\times10^{9}\,\mathrm{N\cdot m^2/C^2}$
speed of light in vacuum	c	$= 3.0\times10^{8}\,\mathrm{m/s}$
Avogadro's number	N_A	$= 6.0\times10^{23}/\mathrm{mol}$
masses of protons and electrons	m_p, m_e	$= 1.7\times10^{-27}\,\mathrm{kg}, 9.1\times10^{-31}\,\mathrm{kg}$

Unit vectors in the direction of the axes $(\vec{e_x}, \vec{e_y}, \vec{e_z})$ or $(\hat{e}_x, \hat{e}_y, \hat{e}_z)$ or $(\hat{\imath}, \hat{\jmath}, \hat{k})$

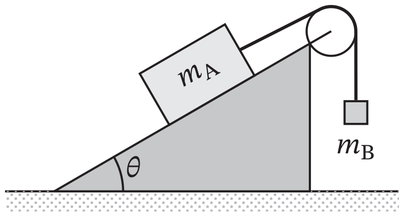
$\vec{E}(\vec{x})$ electric field at $\vec{x}$ $\quad$ $\vec{B}(\vec{x})$ magnetic field (magnetic flux density) at $\vec{x}$
$V(\vec{x})$ electrostatic potential at $\vec{x}$

$\sqrt{2}\approx 1.414 \quad \sqrt{3}\approx 1.732 \quad \sqrt{5}\approx 2.236 \quad \sqrt{7}\approx 2.646 \quad \pi\approx 3.142 \quad \mathrm{e}\approx 2.718$

Answer all the problems.

[Part A] Forces and Energy (16 points)

In **[Part A]**, you may use $g = 1.0 \times 10\,\mathrm{m/s^2}$ as the standard acceleration of gravity (instead of $9.8\,\mathrm{m/s^2}$) for simplicity. Also, use $\cos 36.9° = 0.800$ and $\sin 36.9° = 0.600$.



As in the right figure, a wedge with an angle $\theta = 36.9°$ is fixed on the ground, Block A is on its slope, and Block B is connected to Block A by a wire through a pulley.

Block A has a mass $m_\mathrm{A} = 3.0\,\mathrm{kg}$ and can move only on the slope. Block B has a mass $m_\mathrm{B} = 2.0\,\mathrm{kg}$ and can move only vertically. Between Block A and the slope, the coefficient of kinetic friction is $\mu_\mathrm{k} = 0.250$ and the static-friction coefficient is $\mu_\mathrm{s} = 0.300$. The wire has a negligible mass and does not stretch. Assume its tension is positive throughout this problem. Friction on the pulley is negligible.

At $t = 0$, Block A is moving up the slope with a speed $v = 1.6\,\mathrm{m/s}$. It slows down (with a uniform acceleration) and stops at $t = 2.0\,\mathrm{s}$. Block B does not reach the ground throughout this process.

(1) Calculate the kinetic energy of Block A at $t = 0$.

(2) Calculate the displacement of Block A from $t = 0$ to $2.0\,\mathrm{s}$.

(3) List all the forces exerted on Block A. For each force, describe **(a)** its direction, **(b)** its magnitude, and **(c)** how much work it does on Block A during the two seconds ($t = 0$ to $2.0\,\mathrm{s}$).

(4) Discuss energy conservation.

[Part B] Mathematical Analysis of Circular Motion (16 points)

The Earth orbits around the Sun. Let's approximate it as uniform circular motion around the Sun with a radius R and an angular speed ω, where we consider the Earth as a mass point with the mass M. In particular, we neglect the Earth's rotation.

We define the coordinates (x, y, and z-axes) so that the Sun is at the origin and the Earth's position is given by $\vec{r}(t) = (R\cos\omega t)\vec{e_x} + (R\sin\omega t)\vec{e_y}$. Answer the following questions under this coordinate system.

(1) Calculate the velocity $\vec{v}(t)$ and the acceleration $\vec{a}(t)$ of the Earth.

(2) Describe the angular velocity $\vec{\omega}(t)$ and the angular acceleration $\vec{\alpha}(t)$ of the Earth.

(3) Find the kinetic energy $K(t)$ of the Earth.

(4) Find the angular momentum $\vec{L}(t)$ of the Earth about the Sun.

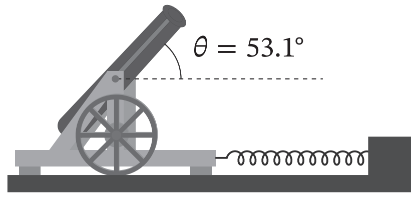
(5) Explain why K does not depend on t.

(6) Explain why $\vec{L}$ does not depend on t.

(7) Discuss momentum conservation.

[The exam questions continue on the next page.]

[Part C] **Momentum** (9 points)



As in the figure, a cannon with a carriage is on the ground, directed rightward. It can move freely in the horizontal direction smoothly (with negligible frictions). It is connected to a post by a large spring with force constant $k = 6.0 \times 10^2\,\mathrm{N/m}$ and equilibrium length $L_0 = 45\,\mathrm{m}$. The mass of the cannon, including its carriage, is $M = 2400\,\mathrm{kg}$.

Initially, the spring is unstretched and the cannon is at rest. At $t = 0$, the cannon fires a projectile, whose mass is 1.0×10^2 kg, with a speed $v_0 = 2.0\times10^2\,\mathrm{m/s}$ in the direction 53.1° above the horizontal to the right. You may use $\cos 53.1° = 0.600$ and $\sin 53.1° = 0.800$.

(1) Find the momentum $\vec{p}_1$ and kinetic energy K_1 of the projectile just after the firing.

(2) Find the momentum $\vec{p}_2$ and kinetic energy K_2 of the cannon just after the firing.

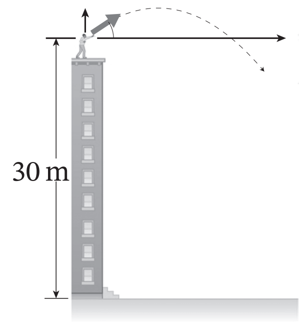
(3) The cannon undergoes oscillatory motion after the fireing. Find the maximum length and minimum length of the spring.

(4) In the previous questions, you should have found $\vec{p}_1 + \vec{p}_2 \neq \vec{0}$. It suggests that the momentum is not conserved during the firing. Discuss why.

[Part D] **Fundamental concepts** (9 points)

(1) From the following physical quantities, select all the quantities that are vector quantities. Please answer by the labels (A, B, etc.), not by the quantity names.

- **(A)** mass
- **(B)** force
- **(C)** speed
- **(D)** velocity
- **(E)** work
- **(F)** power
- **(G)** kinetic energy
- **(H)** temperature
- **(I)** force constant of a spring
- **(J)** constant of kinetic friction
- **(K)** linear momentum
- **(L)** angular momentum about the origin
- **(M)** torque about the origin
- **(N)** moment of inertia of a rigid object
- **(O)** elastic potential energy of a spring–block system
- **(P)** gravitational potential energy of a particle–Earth system



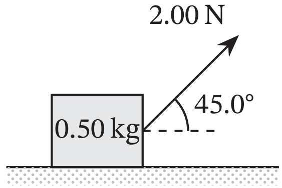
(2) As in the right figure, a stone is thrown from the top of a building upward at an angle $\theta_i = 30.0°$ to the horizontal with an initial speed $v_i = 14\,\mathrm{m/s}$. The height from which the stone is thrown is 30 m above the ground. Assume air resistance is negligible.

What is the speed of the stone just before it strikes the ground?

(3) A 0.50-kg block is on a horizontal surface and initially at rest. The surface have a kinetic friction constant $\mu_k = 0.10$ and a static friction constant $\mu_s = 0.15$.

A boy wants to drag the block, so he applies a constant force of 2.00 N directed rightward with an angle 45.0° above the horizontal, as in the right figure. The block then moves horizontally.

Calculate the kinetic energy of the block at the moment when it has moved 1.00 m from the initial position.



[The exam questions continue on the next page.]

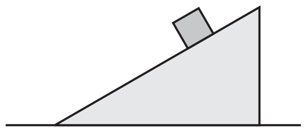
[Part E] **Extra Problem** (unlimited points)

> This is an extra challenging problem, but make an attempt to write something for a partial mark. You are still a student, so no penalty for wrong answers!

As in the right figure, a three-year-old girl puts a wedge on a horizontal floor. Then she puts a tiny block on its slope and releases it gently.

The surfaces of the floor and the wedge slope are flat, smooth, and frictionless. Consider the block as a point mass, disregarding its dimensions, and notice the wedge is not fixed to the floor.

Discuss what will happen quantitatively with paying attention to energy and momentum.



[This is the end of the exam questions.]